Ensemble Kalman inversion for tomographic imaging

Marco Iglesias

School of Mathematical Sciences
University of Nottingham

Workshop on Recent Advances in Iterative Reconstruction,
UCL, May 22nd 2023.

University of Nottingham
UK | CHINA | MALAYSIA

Outline

Outline

Historical Context.

Predictive model for the state v_n (at time t_n):

$$\text{State:} \quad v_{n+1} = f(v_n),$$
$$\text{Observations:} \quad y_{n+1} = H v_{n+1} + \eta_{n+1}.$$

Random initial state v_0 and noise η_n.

Data Assimilation: Given observations $Y_n^\dagger = \{y_\ell^\dagger\}_{\ell=1}^n$ find $v_n | Y_n^\dagger$

Historical Context.

Predictive model for the state v_n (at time t_n):

$$\text{State:} \quad v_{n+1} = f(v_n),$$
$$\text{Observations:} \quad y_{n+1} = Hv_{n+1} + \eta_{n+1}.$$

Random initial state v_0 and noise η_n.

Data Assimilation: Given observations $Y_n^\dagger = \{y_\ell^\dagger\}_{\ell=1}^n$ find $v_n | Y_n^\dagger$

Example: Suppose $f(v_n) = Fv_n$, $v_0 \sim N(m_0, C_0)$ and $\eta_n \sim N(0, \Gamma)$.

$$\text{Then} \qquad v_n | Y_n^\dagger \sim N(m_n, C_n)$$

Historical Context.

Predictive model for the state v_n (at time t_n):

$$\text{State:} \quad v_{n+1} = f(v_n),$$
$$\text{Observations:} \quad y_{n+1} = H v_{n+1} + \eta_{n+1}.$$

Random initial state v_0 and noise η_n.

Data Assimilation: Given observations $Y_n^\dagger = \{y_\ell^\dagger\}_{\ell=1}^n$ find $v_n | Y_n^\dagger$

Example: Suppose $f(v_n) = F v_n$, $v_0 \sim N(m_0, C_0)$ and $\eta_n \sim N(0, \Gamma)$.

$$\text{Then} \qquad v_n | Y_n^\dagger \sim N(m_n, C_n)$$

Kalman filter [Kalman, 1960]

Forecast Step: $\hat{m}_{n+1} = F m_n, \qquad \hat{C}_{n+1} = F C_{n+1} F^T$

Analysis Step:

$$m_{n+1} = \hat{m}_{n+1} + \hat{C}_{n+1} H^T (H \hat{C}_{n+1} H^T + \Gamma)^{-1} (y_{n+1}^\dagger - H \hat{m}_{n+1})$$
$$C_{n+1} = \hat{C}_{n+1} + \hat{C}_{n+1} H^T (H \hat{C}_{n+1} H^T + \Gamma)^{-1} H \hat{C}_{n+1}$$

Nonlinear case: Take $F = Df(v_n)$: Extended Kalman Filter.

Issues for DA: Stability of F and the size of C_n.

Evensen's Ensemble Kalman Filter (EnKF).

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 99, NO. C5, PAGES 10,143–10,162, MAY 15, 1994

Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics

Geir Evensen
Nansen Environmental and Remote Sensing Center, Bergen, Norway

Ensemble Kalman filter (EnKF)

Ensemble of J particles $\{v_n^{(j)}\}_{j=1}^J$ starting with $v_0^{(j)} \sim N(m_0, C_0)$.

$$\text{Forecast Step:} \quad \hat{v}_{n+1}^{(j)} = F v_n^{(j)}$$

Compute empirical mean and covariance

$$\hat{v}_{n+1} = \frac{1}{J}\sum_{j=1}^{J-1} \hat{v}_n^{(j)}, \quad \hat{C}_{n+1} = \frac{1}{J}\sum_{j=1}^{J-1} (\hat{v}_n^{(j)} - \overline{v}_n)(\hat{v}_n^{(j)} - \overline{v}_n)^T$$

$$\text{Analysis Step:} \quad v_{n+1}^{(j)} = \hat{v}_{n+1}^{(j)} + \hat{C}_{n+1} H^T (H \hat{C}_{n+1} H^T + \Gamma)^{-1} (y_{n+1}^{\dagger} + \eta_{n+1}^{(j)} - H \hat{m}_{n+1})$$

where $\eta_{n+1}^{(j)} \sim N(0, \Gamma)$.

$$v_{n+1}^{(j)} \sim N(m_{n+1}, C_{n+1}) = \mathbb{P}(v_{n+1} | Y_{n+1}^{\dagger})$$

Evensen's Ensemble Kalman Filter (EnKF).

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 99, NO. C5, PAGES 10,143–10,162, MAY 15, 1994

Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics

Geir Evensen
Nansen Environmental and Remote Sensing Center, Bergen, Norway

Ensemble Kalman filter (EnKF)

Ensemble of J particles $\{v_n^{(j)}\}_{j=1}^J$ starting with $v_0^{(j)} \sim N(m_0, C_0)$.

Forecast Step: $\hat{v}_{n+1}^{(j)} = f(v_n^{(j)})$

Compute empirical mean and covariance

$$\hat{v}_{n+1} = \frac{1}{J}\sum_{j=1}^{J-1} \hat{v}_n^{(j)}, \quad \hat{C}_{n+1} = \frac{1}{J}\sum_{j=1}^{J-1}(\hat{v}_n^{(j)} - \overline{v}_n)(\hat{v}_n^{(j)} - \overline{v}_n)^T$$

Analysis Step: $v_{n+1}^{(j)} = \hat{v}_{n+1}^{(j)} + \hat{C}_{n+1}H^T(H\hat{C}_{n+1}H^T + \Gamma)^{-1}(y_{n+1}^{\dagger} + \eta_{n+1}^{(j)} - H\hat{m}_{n+1})$

where $\eta_{n+1}^{(j)} \sim N(0, \Gamma)$.

For the nonlinear case: $v_{n+1}^{(j)} \sim N(m_{n+1}, C_{n+1}) \neq \mathbb{P}(v_{n+1}|Y_{n+1}^{\dagger})$

EnKF in Petroleum Engineering

SPE 84372

Reservoir Monitoring and Continuous Model Updating Using Ensemble Kalman Filter

Geir Nævdal, RF-Rogaland Research; Liv Merethe Johnsen, SPE, Norsk Hydro; Sigurd Ivar Aanonsen*, SPE, Norsk Hydro; Erlend H. Vefring, SPE, RF-Rogaland Research

EnKF in Petroleum Engineering

SPE 84372

Reservoir Monitoring and Continuous Model Updating Using Ensemble Kalman Filter

Geir Nævdal, RF-Rogaland Research; Liv Merethe Johnsen, SPE, Norsk Hydro; Sigurd Ivar Aanonsen*, SPE, Norsk Hydro; Erlend H. Vefring, SPE, RF-Rogaland Research

Predictive model:

$$\text{State:} \quad v_{n+1} = f(v_n, u),$$
$$\text{Observations:} \quad y_{n+1} = H v_{n+1} + \eta_{n+1}.$$

Take $z_n = [u_n, v_n]$, $\hat{f}(z_n) = (u_n, f(v_n, u_n))$ and $\hat{H} = (0, H)$.

EnKF in Petroleum Engineering

SPE 84372

Reservoir Monitoring and Continuous Model Updating Using Ensemble Kalman Filter

Geir Nævdal, RF-Rogaland Research; Liv Merethe Johnsen, SPE, Norsk Hydro; Sigurd Ivar Aanonsen*, SPE, Norsk Hydro; Erlend H. Vefring, SPE, RF-Rogaland Research

Predictive model:

$$\begin{aligned} \text{State:} \quad v_{n+1} &= f(v_n, u), \\ \text{Observations:} \quad y_{n+1} &= H v_{n+1} + \eta_{n+1}. \end{aligned}$$

Take $z_n = [u_n, v_n]$, $\hat{f}(z_n) = (u_n, f(v_n, u_n))$ and $\hat{H} = (0, H)$.

EnKF for joint state and parameter estimation:

$$\begin{aligned} \text{State:} \quad z_{n+1} &= \hat{f}(z_n), \\ \text{Observations:} \quad y_{n+1} &= \hat{H} z_{n+1} + \eta_{n+1}. \end{aligned}$$

Particles $z_n^{(j)} = [u_n^{(j)}, v_n^{(j)}]$. Parameter estimate: $\overline{u}_n = \frac{1}{J}\sum_{j=1}^{J} u_n^{(j)}$

Ensemble Kalman Smoother (EnKS)

Assimilation of all data at once

SPE 109808

An Iterative Ensemble Kalman Filter for Data Assimilation

Gaoming Li, SPE, U. of Tulsa and A. C. Reynolds, SPE, U. of Tulsa

Copyright 2007, Society of Petroleum Engineers

An Ensemble Smoother for assisted History Matching $

J.-A.. -A. Skjervheim; G.. Evensen; J.. Hove; J. G. Vabø

Paper presented at the SPE Reservoir Simulation Symposium, The Woodlands, Texas, USA, February 2011.

Paper Number: SPE-141929-MS

https://doi.org/10.2118/141929-MS

Published: February 21 2011

Math Geosci (2012) 44:1–26
DOI 10.1007/s11004-011-9376-z

Ensemble Randomized Maximum Likelihood Method as an Iterative Ensemble Smoother

Yan Chen · Dean S. Oliver

EnKF for PDE-constrained inverse problems

EnKF for PDE-constrained inverse problems

Consider $\mathcal{G} : X \to Y$ the parameter-to-output map.

Parameter Identification

Find u given $y = \mathcal{G}(u) + \eta$.

Classical approach (LSQ):

$$u^* = \arg\min_{u \in \mathcal{A}} \|y - \mathcal{G}(u))\|^2$$

IOP PUBLISHING INVERSE PROBLEMS

Inverse Problems **29** (2013) 045001 (20pp) doi:10.1088/0266-5611/29/4/045001

Ensemble Kalman methods for inverse problems

Marco A Iglesias, Kody J H Law and Andrew M Stuart

Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

EnKF for PDE-constrained inverse problems

Consider $\mathcal{G} : X \to Y$ the parameter-to-output map.

Parameter Identification

Find u given $y = \mathcal{G}(u) + \eta$.

Classical approach (LSQ):

$$u^* = \arg\min_{u \in \mathcal{A}} \|y - \mathcal{G}(u))\|^2$$

IOP PUBLISHING INVERSE PROBLEMS

Inverse Problems **29** (2013) 045001 (20pp) doi:10.1088/0266-5611/29/4/045001

Ensemble Kalman methods for inverse problems

Marco A Iglesias, Kody J H Law and Andrew M Stuart

Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

For $z = (u, \xi) \in X \times Y$,
Define: $f(z) = [u, \mathcal{G}(u)]$,
$H = [0, I]$. Then, $y = Hz + \eta$

Apply EnKF with $Y_n^\dagger = \{y\}_{\ell=1}^n$ and

State: $z_{n+1} = f(z_n)$

Obs: $y_{n+1} = Hz_{n+1} + \eta_{n+1}$

EnKF for PDE-constrained inverse problems

Consider $\mathcal{G} : X \to Y$ the parameter-to-output map.

Parameter Identification

Find u given $y = \mathcal{G}(u) + \eta$.

Classical approach (LSQ):

$$u^* = \arg\min_{u \in \mathcal{A}} \|y - \mathcal{G}(u))\|^2$$

IOP PUBLISHING INVERSE PROBLEMS

Inverse Problems **29** (2013) 045001 (20pp) doi:10.1088/0266-5611/29/4/045001

Ensemble Kalman methods for inverse problems

Marco A Iglesias, Kody J H Law and Andrew M Stuart

Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

For $z = (u, \xi) \in X \times Y$,
Define: $f(z) = [u, \mathcal{G}(u)]$,
$H = [0, I]$. Then, $y = Hz + \eta$

Apply EnKF with $Y_n^\dagger = \{y\}_{\ell=1}^n$ and

$$\text{State:} \quad z_{n+1} = f(z_n)$$

$$\text{Obs:} \quad y_{n+1} = Hz_{n+1} + \eta_{n+1}$$

Algorithm:

$$u_{n+1}^{(j)} = u_n^{(j)} + C_n^{u\mathcal{G}}(C_n^{\mathcal{G}\mathcal{G}} + \Gamma)^{-1}(y - \mathcal{G}(u_n^{(j)}) + \xi_n^{(j)})$$

$$C_n^{\mathcal{G}\mathcal{G}} \equiv \frac{1}{J-1}\sum_{j=1}^{J}(\mathcal{G}(u_n^{(j)}) - \overline{\mathcal{G}}_n)(\mathcal{G}(u_n^{(j)}) - \overline{\mathcal{G}}_n)^T, \quad C_n^{u\mathcal{G}} \equiv \frac{1}{J-1}\sum_{j=1}^{J}(u_n^{(j)} - \overline{u}_n)(\mathcal{G}(u_n^{(j)}) - \overline{\mathcal{G}}_n)^T$$

Conclusion: Ensemble mean seem to solve LSQ with $\mathcal{A} = span\{u_0^{(j)}\}_{j=1}^J$.

Links with iterative regularisation

$$u_{n+1}^{(j)} = u_n^{(j)} + C_n^{u\mathcal{G}}(C_n^{\mathcal{G}\mathcal{G}} + \alpha_n \Gamma)^{-1}(d^\eta - \mathcal{G}(u_n^{(j)}) + \sqrt{\alpha_n}\xi_n^{(j)})$$

α_n: Regularisation parameter.

Informally, as $J \to \infty$,

$$\overline{u}_n = \frac{1}{J}\sum_{j=1}^{J} u_n^{(j)} \to m_n$$

IOP Publishing Inverse Problems

Inverse Problems **32** (2016) 025002 (45pp) doi:10.1088/0266-5611/32/2/025002

A regularizing iterative ensemble Kalman method for PDE-constrained inverse problems

Marco A Iglesias

School of Mathematical Sciences, The University of Nottingham, University Park, Nottingham, NG7 2RD, UK

Links with iterative regularisation

$$u_{n+1}^{(j)} = u_n^{(j)} + C_n^{u\mathcal{G}}(C_n^{\mathcal{G}\mathcal{G}} + \alpha_n \Gamma)^{-1}(d^\eta - \mathcal{G}(u_n^{(j)}) + \sqrt{\alpha_n}\xi_n^{(j)})$$

α_n: Regularisation parameter.

Informally, as $J \to \infty$,

$$\overline{u}_n = \frac{1}{J}\sum_{j=1}^{J} u_n^{(j)} \to m_n$$

IOP Publishing Inverse Problems

Inverse Problems **32** (2016) 025002 (45pp) doi:10.1088/0266-5611/32/2/025002

A regularizing iterative ensemble Kalman method for PDE-constrained inverse problems

Marco A Iglesias

School of Mathematical Sciences, The University of Nottingham, University Park, Nottingham, NG7 2RD, UK

Levenberg-Marquardt (LM)

$$m_{n+1} = m_n + C_n D\mathcal{G}(m_n)^*(D\mathcal{G}(m_n) C_n\, D\mathcal{G}(m_n)^* + \alpha_n \Gamma)^{-1}(y - \mathcal{G}(m_n))$$

or $m_{n+1} = m_n + \Delta m$ with

$$J(\Delta m) = \left\| \Gamma^{-1/2}\Big((y - \mathcal{G}(m_n) - D\mathcal{G}(m_n)\Delta m\Big) \right\|^2 + \alpha_n \left\| C_n^{-1/2} \Delta m \right\|^2 \to \min$$

Links with iterative regularisation

$$u_{n+1}^{(j)} = u_n^{(j)} + C_n^{u\mathcal{G}}(C_n^{\mathcal{G}\mathcal{G}} + \alpha_n \Gamma)^{-1}(d^\eta - \mathcal{G}(u_n^{(j)}) + \sqrt{\alpha_n}\xi_n^{(j)})$$

α_n: Regularisation parameter.

Informally, as $J \to \infty$,

$$\overline{u}_n = \frac{1}{J}\sum_{j=1}^{J} u_n^{(j)} \to m_n$$

IOP Publishing Inverse Problems

Inverse Problems **32** (2016) 025002 (45pp) doi:10.1088/0266-5611/32/2/025002

A regularizing iterative ensemble Kalman method for PDE-constrained inverse problems

Marco A Iglesias

School of Mathematical Sciences, The University of Nottingham, University Park, Nottingham, NG7 2RD, UK

Levenberg-Marquardt (LM)

$$m_{n+1} = m_n + C_n D\mathcal{G}(m_n)^*(D\mathcal{G}(m_n)C_n\, D\mathcal{G}(m_n)^* + \alpha_n \Gamma)^{-1}(y - \mathcal{G}(m_n))$$

or $m_{n+1} = m_n + \Delta m$ with

$$J(\Delta m) = \left\|\Gamma^{-1/2}\Big((y - \mathcal{G}(m_n) - D\mathcal{G}(m_n)\Delta m\Big)\right\|^2 + \alpha_n \left\|C_n^{-1/2}\Delta m\right\|^2 \to \min$$

How to choose α_n?

How to stop?

We borrow the strategy from Hanke's LM

Inverse Problems **13** (1997) 79–95. Printed in the UK PII: S0266-5611(97)77122-3

A regularizing Levenberg–Marquardt scheme, with applications to inverse groundwater filtration problems

Martin Hanke†

Fachbereich Mathematik, Universität Kaiserslautern, D–67653 Kaiserslautern, Germany

Rigorous links with regularisation (**Linear Case**)

Numerische Mathematik (2022) 152:371–409
https://doi.org/10.1007/s00211-022-01314-y

Numerische Mathematik

Check for updates

On convergence rates of adaptive ensemble Kalman inversion for linear ill-posed problems

Fabian Parzer[1] · Otmar Scherzer[1,2,3]

Rigorous links with regularisation (**Linear Case**)

Numerische Mathematik (2022) 152:371–409
https://doi.org/10.1007/s00211-022-01314-y
Numerische Mathematik

Check for updates

On convergence rates of adaptive ensemble Kalman inversion for linear ill-posed problems

Fabian Parzer[1] · Otmar Scherzer[1,2,3]

Continuous time-limit

Take $\alpha_n^{-1} = h \to 0$. System of SDEs for the particles, $u^{(j)}(t)$

SIAM J. NUMER. ANAL.
Vol. 55, No. 3, pp. 1264–1290

ANALYSIS OF THE ENSEMBLE KALMAN FILTER FOR INVERSE PROBLEMS*

CLAUDIA SCHILLINGS† AND ANDREW M. STUART†

In the linear case, particles converge to

$$\overline{u}^* = \arg\min_{u \in \mathcal{A}} ||\Gamma^{-1/2}(y - \mathcal{G}(u))||^2 \qquad t \to \infty$$

EKI as optimiser with different loss function

IOP Publishing Inverse Problems

Inverse Problems **35** (2019) 095005 (35pp) https://doi.org/10.1088/1361-6420/ab1c3a

Ensemble Kalman inversion: a derivative-free technique for machine learning tasks

Nikola B Kovachki and Andrew M Stuart

EKI as optimiser with different loss function

IOP Publishing — Inverse Problems

Inverse Problems **35** (2019) 095005 (35pp) https://doi.org/10.1088/1361-6420/ab1c3a

Ensemble Kalman inversion: a derivative-free technique for machine learning tasks

Nikola B Kovachki and Andrew M Stuart

EKI for Tikhonov

SIAM J. NUMER. ANAL.
Vol. 58, No. 2, pp. 1263–1294

TIKHONOV REGULARIZATION WITHIN ENSEMBLE KALMAN INVERSION*

NEIL K. CHADA†, ANDREW M. STUART‡, AND XIN T. TONG§

No there is no convergence theory for EKI when $\mathcal{G}$ is nonlinear!!

When to use EKI

- Computationally costly nonlinear forward map $\mathcal{G} : X \to Y$ defined in black-box fashion on a very large X.
- Problems where there is an underlying distribution for the unknown (for the prior ensemble). Problem needs to be suitably parameterised.
- For low-dimensional problems EKI only makes sense if $\mathcal{G}$ is computationally very costly.

Outline

Example: EKI for Ground Penetrating Radar

Work done in collaboration with:

Rania Patsia, Antonis Giannopoulos, Nick Polydorides

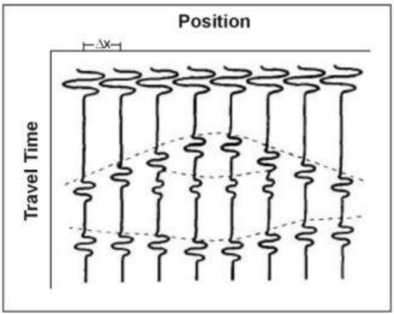
School of Engineering, University of Edinburgh.

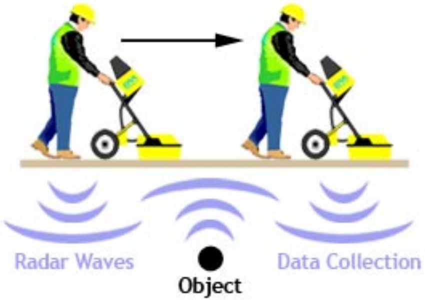
Example: EKI for Ground Penetrating Radar

Work done in collaboration with:

Rania Patsia, Antonis Giannopoulos, Nick Polydorides
School of Engineering, University of Edinburgh.

Applications: Geophysics, Archeology, Defence, Civil Engineering.

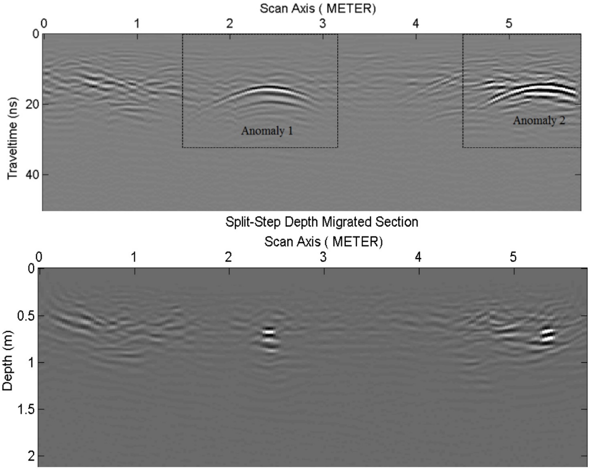




Pictures from:
https://archive.epa.gov/esd/archive-geophysics/web/html/ground-penetrating_radar.html
http://www.worksmartinc.net

Practical approach for GPR data

Data migration



From *https://doi.org/10.1051/e3sconf/20183503004*

Limitations: Unrealistic modelling assumptions (i.e. constant velocity), no measures of uncertainty.

Full-Waveform inversion.

Maxwell equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \qquad \nabla \cdot \epsilon \mathbf{E} = 0$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_s, \qquad \nabla \cdot \mu \mathbf{H} = 0$$

Forward map

$$u = (\epsilon, \mu, \sigma) \longrightarrow \mathcal{G}(u) = \{V_r(t) \propto E_y(\mathbf{x}_r, t)\}_{r=1}^{R}$$

Aim: Infer $u^\dagger$ given measurements, y, of $\mathcal{G}(u^\dagger)$.
Existing Approaches:

Deterministic: $\|y - \mathcal{G}(u)\|^2 + \mathcal{R}(u) \to \min$

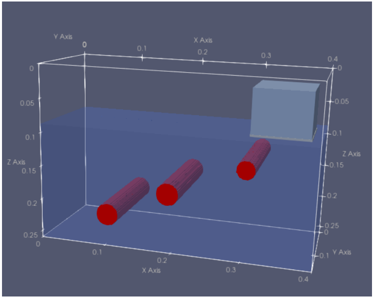
with some choice of regulariser $\mathcal{R}(u)$.

Bayesian (MAP): $\mathbb{P}(u|y) \propto \mathbb{P}(y - \mathcal{G}(u))\mathbb{P}(u)$

for some prior $\mathbb{P}(u)$.

High-contrast targets

Targets that will induce a high contrast in material properties



Crucial: choice of $\mathcal{R}(u)$ (or $\mathbb{P}(u)$), e.g. edge-preserving regulariser/prior.

Shape reconstruction of Perfect Electric Conductor (PEC) targets.

Incorporate the target region via Maxwell Eqs' boundary conditions:

PEC Boundary Conditions:

$$\mathbf{n} \times \mathbf{E} = 0, \qquad \mathbf{n} \cdot \mathbf{H} = 0, \qquad \text{on} \quad \partial\Omega$$

where $\partial\Omega$ is the boundary of the PEC region Ω.

Shape Inversion: Infer Ω given measurements, y, of $\{V_r(t)\}_{r=1}^{R} = \mathcal{G}(\Omega)$.

Shape reconstruction of Perfect Electric Conductor (PEC) targets.

Incorporate the target region via Maxwell Eqs' boundary conditions:

PEC Boundary Conditions:

$$\mathbf{n} \times \mathbf{E} = 0, \qquad \mathbf{n} \cdot \mathbf{H} = 0, \qquad \text{on} \quad \partial\Omega$$

where $\partial\Omega$ is the boundary of the PEC region Ω.

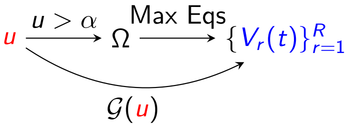
Shape Inversion: Infer Ω given measurements, y, of $\{V_r(t)\}_{r=1}^{R} = \mathcal{G}(\Omega)$.

Level-set parameterisations of targets

We introduce an artificial *level-set function* $u : \mathbb{R}^3 \to \mathbb{R}$ and define the target via

$$\Omega = \left\{ \mathbf{x} \in \mathbb{R}^3 \middle| u(x) > \alpha \right\}$$

Forward map:

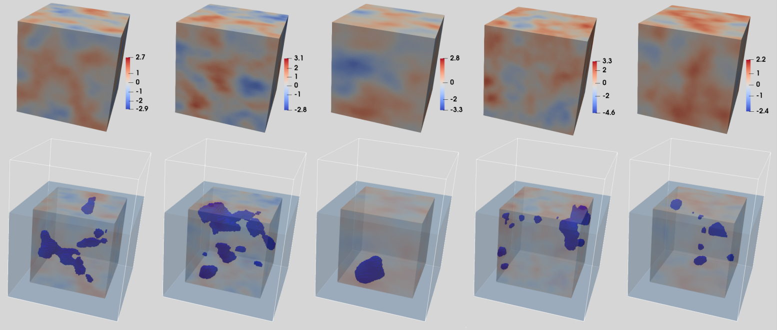


Use EKI to infer an optimal level-set function

Gaussian random fields (top) for the initial ensemble

$$u_0^{(j)} \sim N(0, \mathcal{C})$$

where $\mathcal{C}$ is a Matérn covariance operator.
Bottom: Regions defined via the truncation of the maps above with $\alpha = 1.75$.



Goal: Apply EKI on the ensemble of level-set functions:

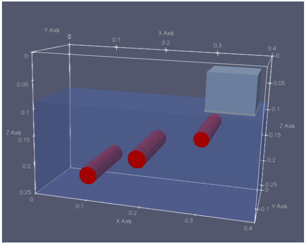
$$u_{n+1}^{(j)} = u_n^{(j)} + C_n^{u\mathcal{G}}(C_n^{\mathcal{G}\mathcal{G}} + \alpha_n \Gamma)^{-1}(y - \mathcal{G}(u_n^{(j)}) + \sqrt{\alpha_n}\xi_n^{(j)})$$

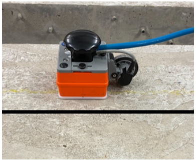
Forward map:

$$u \xrightarrow{u > \alpha} \Omega \xrightarrow{\text{gprMax}} \{V_r(t)\}_{r=1}^{R}$$

$$u \xrightarrow{\mathcal{G}(u)} \{V_r(t)\}_{r=1}^{R}$$

gprMax: FDTM with built-in GSSI antenna model.
We assume targets are immersed in concrete (dispersive media).





Synthetic Experiment I

Simulation domain: 20cm x 20cm x 25cm.

Time window: 4ns

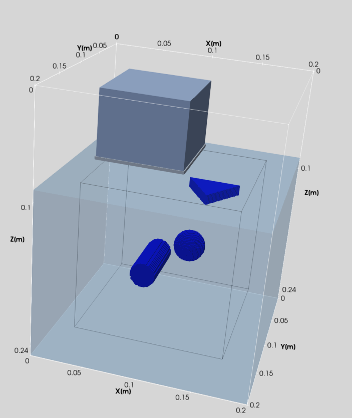
No. Cells: 200 x 200x 250$= 10^7$

Concrete domain: : 20cm x 20cm x 16.3cm

Measurements: 16 traces (spaced by 2.5cm)

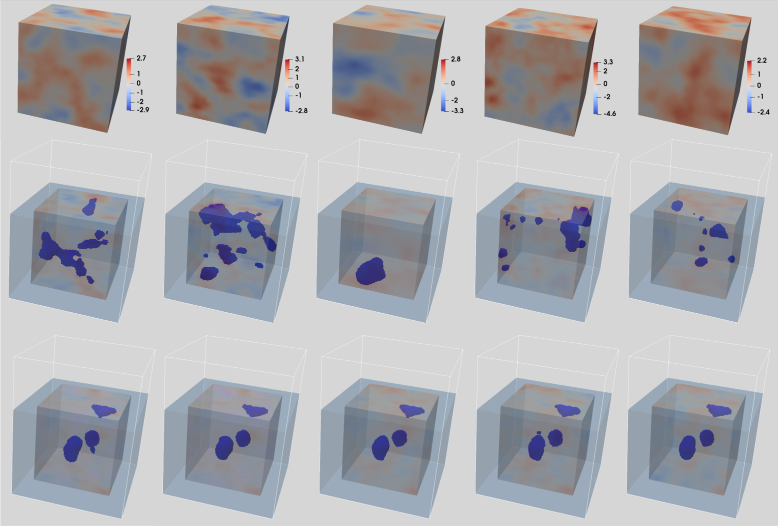
We use gprMax to simulate 16 traces and we corrupt them with 2%.

We apply EKI with $J = 10^3$ samples.

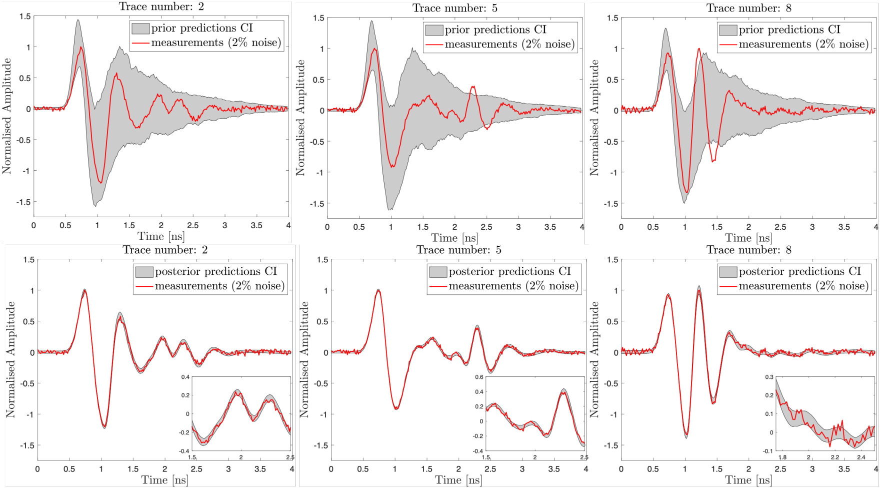


EKI samples

Top: Prior LS function. Middle: Prior LS function + region.
Bottom: posterior LS function + region.

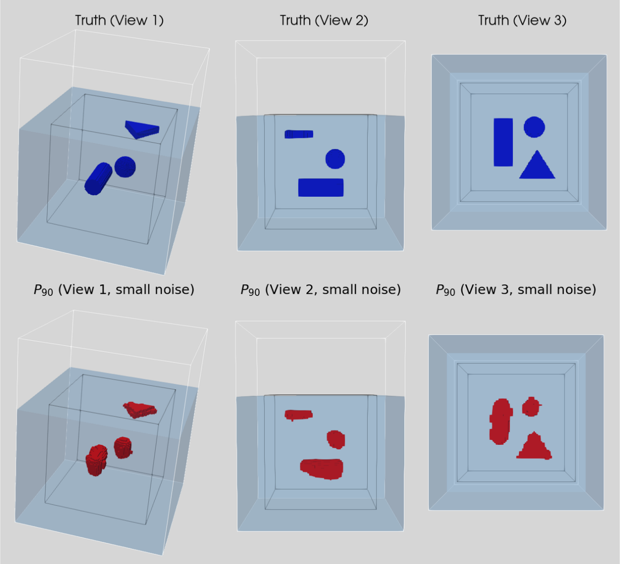


Fit to the measurements



EKI Posterior Estimates

Probability for taget: $P(\mathbf{x}) = \mathbb{P}(u(\mathbf{x}) > \alpha)$



Computational Cost (Sulis -Tier 2 Midlands Plus)

Prediction step using 3-traces simulation runs (gprMax).
EKI algorithm needs between 10 and 20 iterations. We use $J = 500$ samples.

Sulis' compute partition

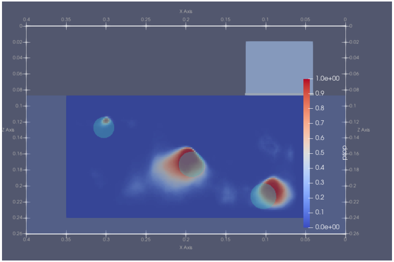
- 1 core $\approx$ 49 mins/ sim. Total time is unfeasible
- 1 node (128 cores) using MPI run 4 tasks (30 cpus) $\approx$ 4.3 mins/sim. Total time $\approx$ 22 days
- 30 full nodes using MPI run on each node. Total time $\approx$ 18 hrs

Sulis' gpu partition

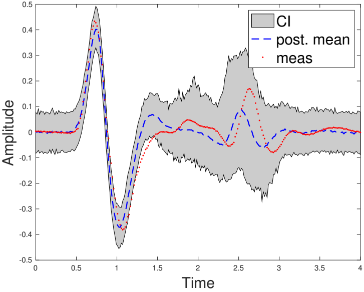
- 1 GPU +MPI (with 15 CPUs each) 0.45 sec/sim. Total time $\approx$ 3.9 days.
- 30 GPUs Total time $\approx$ 3.1 hrs.

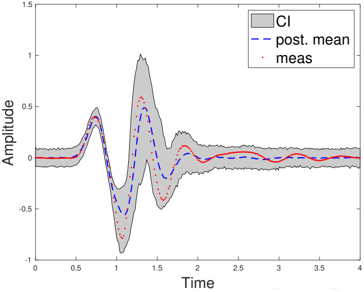
C. Warren, A. Giannopoulos, A. Gray, I. Giannakis, A, Patterson, L. Wetter, A. Hamrah
A CUDA-based GPU engine for gprMax: Open source FDTD electromagnetic simulation software
Computer Physics Communications. 237 (2019) 208-218

Experiment with real data.



Prediction of traces (we need to inflate error covariance)





EKI for Bayesian Inverse Problems

OPEN ACCESS
IOP Publishing — Inverse Problems
Inverse Problems **37** (2021) 025008 (40pp) https://doi.org/10.1088/1361-6420/abd29b

Adaptive regularisation for ensemble Kalman inversion

Marco Iglesias* **and Yuchen Yang**

Recall

$$y = \mathcal{G}(u) + \eta$$

Bayes' rule:

$$\mathbb{P}(u|y) \propto \exp\left[-\frac{1}{2}||\Gamma^{-1/2}(y-\mathcal{G}(u))||^2\right]\mathbb{P}(u)$$

Tempering:

Introduce $\{\Phi_n\}_{n=0}^{N}$ such that $0 = \Phi_0 < \Phi_1 \cdots < \Phi_{N+1} = 1$ and define

$$\mathbb{P}_{n+1}(u) \propto \mathbb{P}_n(u) \exp\left[-\frac{1}{2}||(\alpha_n\Gamma)^{-1/2}(y-\mathcal{G}(u))||^2\right]$$

$\alpha_n = [\Phi_n - \Phi_{n-1}]^{-1}$. (Note that $\sum_{n=1}^{N} \alpha_n^{-1} = 1$)

Suppose $\pi_0 = N(m_0, C_0)$ is a Gaussian approximation.

$$\pi_{n+1}(u) \propto \pi_n(u) \exp\left[-\frac{1}{2}||(\alpha_n\Gamma)^{-1/2}(y-\mathcal{G}(m_n) - D\mathcal{G}(m_n)(u-m_n))||^2\right]$$

EKI will produce a ensemble of Gaussian approximations to $\pi_{n+1}(u)$

Ensemble Kalman Sampling (EKS)

SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 19, No. 1, pp. 412–441

Interacting Langevin Diffusions: Gradient Structure and Ensemble Kalman Sampler*

Alfredo Garbuno-Inigo[†], Franca Hoffmann[†], Wuchen Li[‡], and Andrew M. Stuart[†]

Ensemble Kalman Sampling (EKS)

SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 19, No. 1, pp. 412–441

© 2020 Society for Industrial and Applied Mathematics

Interacting Langevin Diffusions: Gradient Structure and Ensemble Kalman Sampler*

Alfredo Garbuno-Inigo†, Franca Hoffmann†, Wuchen Li‡, and Andrew M. Stuart†

Mean-Field

Ensemble Kalman Methods: A Mean Field Perspective

Edoardo Calvello*, Sebastian Reich†, and Andrew M. Stuart‡

Ensemble Kalman Sampling (EKS)

SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 19, No. 1, pp. 412–441

© 2020 Society for Industrial and Applied Mathematics

Interacting Langevin Diffusions: Gradient Structure and Ensemble Kalman Sampler*

Alfredo Garbuno-Inigo†, Franca Hoffmann†, Wuchen Li‡, and Andrew M. Stuart†

Mean-Field

Ensemble Kalman Methods: A Mean Field Perspective

Edoardo Calvello*, Sebastian Reich†, and Andrew M. Stuart‡

No there is no convergence theory for EKI or EKS!!

Outline

Additional examples of EKI+level-set: Resin Infusion of Reinforcements

Composites: Part A 143 (2021) 106323

Contents lists available at ScienceDirect

Composites Part A

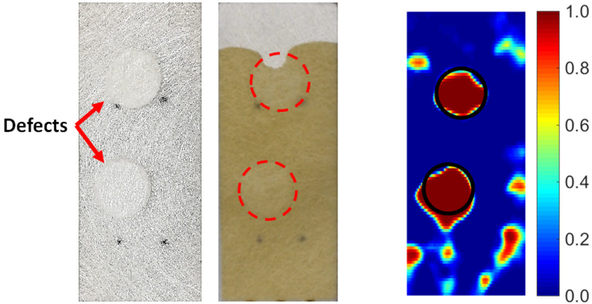
journal homepage: www.elsevier.com/locate/compositesa

ELSEVIER

Bayesian inversion algorithm for estimating local variations in permeability and porosity of reinforcements using experimental data

M.Y. Matveev [a,*], A. Endruweit [a], A.C. Long [a], M.A. Iglesias [b], M.V. Tretyakov [b]

[a] Composites Research Group, Faculty of Engineering, University of Nottingham, UK
[b] School of Mathematical Sciences, University of Nottingham, UK



EKI+level-set: Electrical Resistivity Tomography

Field cross borehole sandstone stratigraphy
Eggborough, UK.

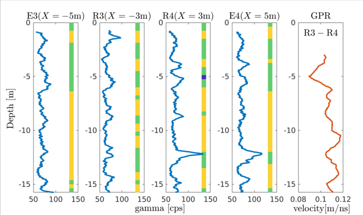
Geophysical Journal International

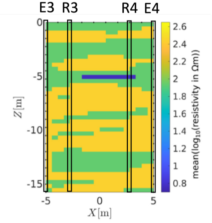
Geophys. J. Int. (2021) **225**, 887–905 doi: 10.1093/gji/ggab013
Advance Access publication 2021 January 11
GJI General Geophysical Methods

Efficient multiscale imaging of subsurface resistivity with uncertainty quantification using ensemble Kalman inversion

Chak-Hau Michael Tso,[1,2] Marco Iglesias,[3] Paul Wilkinson,[4] Oliver Kuras,[4] Jonathan Chambers[4] and Andrew Binley[2]

[1]Environmental Data Science Team, UK Centre for Ecology and Hydrology, Lancaster LA1 4AP, UK. E-mail: mtso@ceh.ac.uk
[2]Lancaster Environment Centre, Lancaster University, Lancaster LA1 4YQ, UK
[3]Department of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK
[4]Geophysical Tomography Team, British Geological Survey, Keyworth NG12 5GG, UK





EKI+level-set: Magnetic Resonance Elastography

Phys. Med. Biol. **67** (2022) 235003 https://doi.org/10.1088/1361-6560/ac9fa1

Physics in Medicine & Biology

IPEM Institute of Physics and Engineering in Medicine

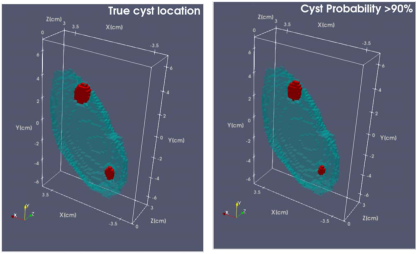
PAPER

Ensemble Kalman inversion for magnetic resonance elastography

Marco Iglesias[1], Deirdre M McGrath[2,3], M V Tretyakov[1] and Susan T Francis[2,3]

[1] School of Mathematical Sciences, University of Nottingham, Nottingham, United Kingdom
[2] NIHR Nottingham Biomedical Research Centre, Nottingham University Hospitals NHS Trust and University of Nottingham, Nottingham, United Kingdom
[3] Sir Peter Mansfield Imaging Centre, University of Nottingham, Nottingham, United Kingdom

Characterisation of elastic properties of biological tissues as well as the presence of malignancies.



Thank you

https://www.maths.nottingham.ac.uk/personal/pmzmi/publications/